EXTENDED RESEARCH STATEMENT

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Introduction

My main area of research is Harmonic Analysis. A significant portion of my work is related to the study of singular integrals in a broad sense and the function spaces related to them. Given a measure space (Ω, μ), a singular integral operator is an operator of the form

$$Tf(x) \sim \int_\Omega K(x,y)f(y)d\mu(y)$$

(the symbol '$\sim$' needs to be understood as '$=$' in some sensible way). The integral **kernel** K is often singular along the diagonal $\{x = y\} \subset \Omega \times \Omega$. An important family of singular integral operators is the class of Calderón-Zygmund operators. These are operators whose kernels (on $\mathbb{R}^d$) behave like

$$K(x,y) \sim \frac{1}{|x-y|^d} \text{ when } x \text{ is very close to } y.$$

In particular, this often implies that the integral defining T is not absolutely convergent. Probably, the simplest example is the Hilbert transform $\mathcal{H}$, an important and useful operator in many areas of Mathematics. It can be defined as follows:

$$\mathcal{H}f(x) = \text{p.v.} \int_{\mathbb{R}} \frac{f(y)}{x-y}\, dy.$$

Other kinds of singular operators are fractional integrals (for which the singularity of the kernel is locally integrable), that appear in the solution of PDE, martingale transforms of Probability theory and Haar shift operators, that are discrete models of Calderón-Zygmund operators.

In my research, I am interested in Calderón-Zygmund operators, martingale transforms or dyadic Haar shifts from a perspective much wider than that of the Euclidean space $\mathbb{R}^d$. This implies that many times I let the underlying measure space (Ω, μ) be very general. Of course, nondoubling measures appear in such an approach –such as the Hausdorff measure of certain fractals–. Also, I am interested in noncommutative forms of (Ω, μ) which lead to the analysis of Calderón-Zygmund operators for matrix-valued functions or noncommutative martingale inequalities in arbitrary von Neumann algebras. The applications of such generalizations of the classical theory point at problems of Theoretical Physics or Free Probability. Precisely, a common theme in my research is the role played by probabilistic techniques in Harmonic Analysis, and also the role that analytic tools can play in Probability theory.

In the following sections I describe in more detail the context of my contributions and the lines of research I plan to pursue. The material is organized in three categories: classical Harmonic Analysis, nondoubling measures and noncommutative L_p spaces and martingales.

1. Classical Harmonic Analysis

The fact that L^p spaces ($1 < p < \infty$) are preserved under the action of Calderón-Zygmund operators is something that can now be considered classical. If one replaces the Lebesgue measure by some weight in the correct class the previous statement remains true, but, of course, the norm of the operator depends on the weight. This has also been known for more than 40 years. However, the problem of estimating the precise quantitative dependence on the weights has attracted much attention in the last decade. Many times, a very useful way to study an operator in Harmonic Analysis is to decompose it into sums of simpler dyadic operators. Using this strategy, Petermichl was able to prove the sharp weighted estimate for the Hilbert transform in L^2 (see [20]). In particular, Petermichl showed that the Hilbert transform satisfies

$$\|\mathcal{H}f\|_{L^2(w)} \lesssim [w]_{A_2} \|f\|_{L^2(w)}, \tag{1.1}$$

for all A_2 weights w. This was a key step towards the full A_2 theorem for general Calderón-Zygmund operators –that is, replacing the Hilbert transform in (1.1) by a general Calderón-Zygmund operator T– which was finally proved by Hytönen in [11]. Approximately two years later, Lerner proved the following result, which is sometimes known as *Lerner domination theorem* [14]:

$$\|Tf\|_{\mathbb{X}} \lesssim \sup_{\mathscr{D},\mathscr{S}} \|\mathcal{A}_{\mathscr{S}} f\|_{\mathbb{X}}, \tag{1.2}$$

where the supremum above is taken over all dyadic grids $\mathscr{D}$ and all **sparse** families $\mathscr{S} \subset \mathscr{D}$, and $\mathbb{X}$ is any Banach space of functions. It turns out that the quantitative behavior of the dyadic operators

$$\mathcal{A}_{\mathscr{S}} f(x) = \sum_{Q \in \mathscr{S}} \langle f \rangle_Q \chi_Q(x)$$

can be described with high precision, which yields a different proof of the A_2 theorem, among other interesting applications. However, the validity of (1.2) in the setting of quasi-Banach spaces –spaces of functions where the triangle inequality of the norm is satisfied in the weaker sense $\|f+g\| \leq C(\|f\| + \|g\|)$– remained as an open problem for some time. Our first main contribution together with G. Rey, answers this on the affirmative by establishing a pointwise bound between the Calderón-Zygmund operator T and a finite amount of sparse operators:

Theorem 1.1 (C., Rey [9])**.** *Fix a Calderón-Zygmund operator T and a function f. There exist $d+1$ dyadic systems $\mathscr{D}^j$, $0 \leq j \leq d$, and $d+1$ sparse operators $\mathcal{A}_{\mathscr{S}^j}$, $\mathscr{S}^j \subset \mathscr{D}^j$ such that*

$$|Tf(x)| \lesssim \sum_{j=0}^{d} \mathcal{A}_{\mathscr{S}^j} |f|(x).$$

In particular, let $\mathbb{X}$ be any quasi-Banach lattice of functions. Then we have the domination theorem

$$\|Tf\|_{\mathbb{X}} \leq C \sup_{\mathscr{D},\mathscr{S}} \|\mathcal{A}_{\mathscr{S}} f\|_{\mathbb{X}}.$$

for any Calderón-Zygmund operator T. The supremum is taken over all dyadic grids $\mathscr{D}$ and all sparse families $\mathscr{S} \subset \mathscr{D}$.

The proof of theorem 1.1 relies on a pointwise formula that relates dyadic shifts of arbitrary complexity and the sparse dyadic shifts $\mathcal{A}_{\mathscr{S}}$. An interesting feature of this pointwise formula is the fact that it is valid regardless of the measure μ with respect to which one constructs the averages in the definition of $\mathcal{A}_{\mathscr{S}}$. However, a generalization of theorem 1.1 to the setting of nondoubling measures requires new ideas and is still partially open (see section 2).

In theorem 1.1, we require certain smoothness on the kernel K of T. In particular, we ask that K satisfies the so-called **logarithmic Dini condition**, which is an integral condition on the modulus of continuity of K. This condition has been relaxed to the milder Dini condition [13, 15]. However, one may consider alternative smoothness conditions, such as integral ones –in the spirit of the Hörmander condition–. In that regard, together with Bui, Duong and Hormozi, we consider the following Hörmander type L^{p_0} condition (for some $1 < p_0 < \infty$):

$$\Big(\int_{S_j(B)} |K(x,y) - K(\overline{x},y)|^{p_0'} dy\Big)^{1/p_0'} \lesssim \frac{|x-\overline{x}|^{\delta - n/p_0}}{|B|^{\delta/n}} 2^{-\delta j_0} \tag{1.3}$$

for some $\delta > d/p_0$, all balls B, all $x, \overline{x} \in \frac{1}{2}B$ and $S_j(B) = 2^j B \backslash 2^{j-1}B$, $j \geq 1$. The result that we obtain is the following:

Theorem 1.2 (Bui, C., Duong, Hormozi [1]). *Fix $p_0 > 1$. If the kernel K satisfies (1.3), we have the pointwise bound*

$$|Tf(x)| \lesssim \sum_{j=0}^{d+1} \mathcal{A}^{p_0}_{\mathscr{S}^j} |f|(x),$$

where

$$\mathcal{A}^{p_0}_{\mathscr{S}} f(x) = \sum_{Q \in \mathscr{S}} \langle |f|^{p_0} \rangle_Q^{1/p_0} \chi_Q(x)$$

The operators $\mathcal{A}^{p_0}_{\mathscr{S}}$ exhibit similar quantitative properties to $\mathcal{A}_{\mathscr{S}}$, yielding the same sharp weighted norm inequalities. It is worth noting that the results in theorems 1.1 and 1.2 both generalize to the multilinear setting. Our results so far are not enough to show what is the mildest smoothness condition that one can ask in order to obtain sharp weighted inequalities or domination by sparse operators. We consider this a very interesting open problem.

Problem 1.3. *What is the minimal smoothness assumption on K so that we have the weighted bound $\|T\|_{L^2(w)} \lesssim [w]_{A_2}$?*

Both in theorem 1.1 and in theorem 1.3 the amount of sparse operators that we need $(d+1)$ is optimal. This is because it is the precise number of dyadic lattices necessary to cover any ball of $\mathbb{R}^d$ by cubes of comparable size:

Theorem 1.4 (C. [4]). *On $\mathbb{R}^d$, there exist $d+1$ dyadic lattices $\mathscr{D}^j$, $0 \leq j \leq d$, such that for each ball B there exists $Q \in \cup_j \mathscr{D}^j$ that satisfies*

$$B \subset Q \subset c_d B,$$

where the constant c_d depends only on d. The constant $d+1$ is the smallest possible so that this holds.

Our proof of theorem 1.4 relies heavily on the Hilbert space structure of $\mathbb{R}^d$ via orthogonal projections. On the other hand, (non-optimal) covering lemmas in the spirit of ours do exist for a wide class of metric spaces. Therefore, it is natural to ask whether there exist optimal coverings such as the one above in more general metric spaces. This can be precisely stated as follows:

Problem 1.5. *Assume $\mathbb{X}$ is a geometrically doubling metric space, hence finite dimensional. What is the minimum number of dyadic lattices (à la Christ [3] or Hytönen-Kairema [12]) that we need to have a covering like that of theorem 1.4?*

2. Nondoubling Measures

A measure μ on a metric space $\mathbb{X}$ is said to be doubling if the following condition holds for all balls $B(x,r)$ centered at x and with radius r:

$$\mu(B(x,2r) \leq C_\mu \mu(B(x,r)). \tag{2.1}$$

If (2.1) does not hold for any constant C_μ, then we say that μ is **nondoubling**. The basic theory of singular integrals acting on spaces equipped with nondoubling measures has been studied since the late 90's by Nazarov, Tolsa, Treil, Volberg and others [18], [21]. The absence of the doubling property led to the use of new techniques in Harmonic Analysis and to the introduction of new function spaces. In particular, it became clear that the definition of the BMO spaces that would serve as endpoints for the boundedness of Calderón-Zygmund operators needed to be modified (see, for example, Verdera's example [23]). This has led to new families of BMO spaces adapted to each class of measures under consideration. One such family appears in the study of measures whose density decays (or grows) very fast, such as the Gaussian measure –the probability measure on $\mathbb{R}^d$ whose density is given by $d\gamma(x) = C_d e^{-|x|^2} dx$–. This class of measures was studied by Carbonaro-Mauceri-Meda [2, 17], where they deal with densities whose growth is at least exponential. However, a different, non-geometric approach is also possible. To that end, consider a general measure space (Ω,μ), and assume that one can find what we call an **admissible covering**. This means that there exists a pair of atomic σ-algebras, $(\Sigma_{\mathrm{a}},\Sigma_{\mathrm{b}})$, such that $\Sigma_{\mathrm{a}} \cap \Sigma_{\mathrm{b}} = \{\Omega,\emptyset\}$ and

$$\min\left\{ \sup_{A\in\Pi_{\mathrm{a}}\setminus\{A_0\}} \sum_{B\in R_A} |R_B| \frac{\mu(A\cap B)^2}{\mu(A)\mu(B)}, \sup_{B\in\Pi_{\mathrm{b}}\setminus\{B_0\}} \sum_{A\in R_B} |R_A| \frac{\mu(A\cap B)^2}{\mu(A)\mu(B)} \right\} < 1$$

(R_A is the set of atoms in Σ_{b} that touch A, and R_B is defined similarly). Then, we have the following result:

Theorem 2.1 (C., Mei, Parcet [6]). *Let (Ω,Σ,μ) be a σ-finite measure space. If there exists an admissible covering of (Ω,μ) by $(\Sigma_{\mathrm{a}},\Sigma_{\mathrm{b}})$, then consider two filtrations of Σ by successive refinement of $\Sigma_{\mathrm{a}},\Sigma_{\mathrm{b}}$. Construct the corresponding martingale* BMO *spaces, and denote their intersection by* $\mathrm{BMO}_{\mathrm{ab}}$. *Then, we have the following interpolation result:*

$$[\mathrm{BMO}_{\mathrm{ab}}, L^1]_{\frac{1}{p}} = L^p.$$

The meaning of theorem 2.1 is that, if and admissible covering exists, we can construct a BMO-type space suitable for the theory of Calderón-Zygmund operators. This is so because, only asking that the atoms in the filtrations that define $\mathrm{BMO}_{\mathrm{ab}}$ be doubling, we immediately obtain that L^2 bounded Calderón-Zygmund operators T satisfy $T : L^\infty \to \mathrm{BMO}_{\mathrm{ab}}$ and hence are L^p bounded, $1 < p < \infty$. On the other hand, by construction we obtain both the John-Nirenberg property and an atomic description of the predual of $\mathrm{BMO}_{\mathrm{ab}}$ (see section 3). On the other hand, our mild conditions on the measure μ ensure that the most important ones considered by Carbonaro-Mauceri-Meda fit in our scheme. Also, we provide new examples which indicate that the class of measures that we consider is, indeed, larger. In particular, measures ν on $\mathbb{R}^d$ whose density decays at a polynomial rate –that is, $d\nu(x) = (1+|x|)^{-s}$, $s \gtrsim d^{3/2}$– admit an admissible covering and hence are under the scope of theorem 2.1.

Our second contribution is related to a different class of measures, namely those of **polynomial growth**. A measure μ on $\mathbb{R}^d$ has n-polynomial growth, $0 < n \leq d$, if for all points x and radii r, we have

$$\mu(B(x,r) \lesssim r^n.$$

Those measures may be nondoubling. Tolsa defined a BMO-type space (see [22]), RBMO, which is satisfactory from the point of view of Calderón-Zygmund theory. Also, by means of a centered Calderón-Zygmund decomposition (see [21]), one can recover the weak $(1,1)$ endpoint estimate for Calderón-Zygmund operators under appropriate conditions on the kernel K. But despite this remarkable results, the dyadic part of the theory was essentially missing, mainly due to the lack of the 'correct' dyadic system. It turns out that such a dyadic system exists, based on a construction by David and Mattila [10]:

Theorem 2.2 (C., Parcet [8])**.** *Let μ be a measure of n-polynomial growth on $\mathbb{R}^d$. Then there exist positive constants $\alpha, \beta > 100$ and a two-sided filtration $\Sigma = \{\Sigma_k : k \in \mathbb{Z}\}$ of atomic σ-algebras of* $\operatorname{supp}(\mu)$ *that satisfy the following properties, where $\Pi(\Sigma)$ denotes the set of atoms in the filtration*:

i) *The σ-algebras Σ_k are increasingly nested.*
ii) *Given any $Q \in \Pi(\Sigma)$, the union of its antecessors covers the whole* $\sup(\mu)$.
iii) *If $Q \in \Pi(\Sigma)$, there exists an (α, β)-doubling ball B_Q with $B_Q \subset Q \subset 28B_Q$.*
iv) *If $x \in Q \in \Pi(\Sigma)$, then*

$$R = \bigcap_{\substack{S \in \Pi(\Sigma) \\ S \supsetneq Q}} S \qquad \Rightarrow \qquad \int_{\alpha B_R \setminus 56 B_Q} \frac{d\mu(y)}{|x-y|^n} \lesssim_{n,d,\alpha,\beta} 1.$$

The system Σ is highly nonregular. However, properties iii) and iv) (doublingness and some sort of 'weak regularity') are all that is required to prove many results in nonhomogeneous Calderón-Zygmund theory. For example, consider the martingale BMO associated to Σ –let us call it $\mathrm{RBMO}_\Sigma(\mu)$–, that is, the space equipped with the norm

$$\|f\|_{\mathrm{RBMO}_\Sigma} = \sup_{k \in \mathbb{Z}} \Big\| \mathsf{E}_{\Sigma_k} \big| f - \mathsf{E}_{\Sigma_{k-1}} f \big|^2 \Big\|_\infty^{\frac12}.$$

Then $\mathrm{RBMO}_\Sigma(\mu)$ is an improvement of Tolsa's RBMO in the following sense:

Theorem 2.3 (C., Parcet [8])**.** *The space* $\mathrm{RBMO}_\Sigma(\mu)$ *satisfies*:

i) $\mathrm{RBMO}(\mu) \subsetneq \mathrm{RBMO}_\Sigma(\mu)$.
ii) *If a Calderón-Zygmund operator T is bounded on $L^2(\mu)$, then*

$$T : L^\infty(\mu) \mapsto \mathrm{RBMO}_\Sigma(\mu).$$

iii) *John-Nirenberg inequality and Fefferman-Stein duality theory.*
iv) *Interpolation in the category of Banach spaces:*

$$[\mathrm{RBMO}_\Sigma(\mu), L^1(\mu)]_{\frac1p} = L^p(\mu) \quad \textit{for} \quad 1 < p < \infty.$$

Other features of the filtration Σ are the existence of a dyadic Calderón-Zygmund decomposition, in the spirit of the one in [16], which can be used to deduce weak $(1,1)$ bounds for both CalderónZygmund operators and their dyadic models. Finally, Σ can also be used to address the subject of pointwise domination of nonhomogeneous Calderón-Zygmund operators. In particular, we have the following result, which is the first of this kind for nondoubling measures:

Theorem 2.4 (C., Parcet [8]). *Given a Calderón-Zygmund operator T, the following pointwise estimate holds for certain sparse (with respect to μ) family $\mathcal{S} \subset \Pi(\Sigma)$:*

$$Tf(x) \lesssim_{d,n,\mu} \sum_{Q \in \mathcal{S}} \left[\inf_{y \in Q} \mathcal{M}^c f(y) \right] \chi_Q(x).$$

In particular, if w is an A_2 weight with respect to μ, we have

$$\big\| T : L_2(w d\mu) \to L_2(w d\mu) \big\| \lesssim_{d,n,\mu} [w]^2_{A_2(\mu)}.$$

The maximal operator $\mathcal{M}^c$ in the statement above is the only obstacle in proving the A_2 linear bound for Calderón-Zygmund operators in the nondoubling setting. This suggests the following problem:

Problem 2.5. *Can one prove theorem 1.1 –or (1.2)– in the setting of measures of polynomial growth?*

Another possible direction of research concerns multiparameter Analysis. Dyadic-probabilistic techniques are crucial in the theory of multiparameter Harmonic Analysis, and hence the space $\mathrm{RBMO}_\Sigma(\mu)$ may be useful in determining the 'right' BMO space in the multiparameter setting.

Problem 2.6. *Determine a satisfactory BMO-type space for multiparameter Calderón-Zygmund theory in the nonhomogeneous setting.*

3. Noncommutative L_p spaces, noncommutative martingales

We consider now problems in certain vector-valued settings. Those come from the realm of noncommutative Harmonic Analysis. In this setting, the noncommutative analogue of a measure space is a pair $(\mathcal{M}, \tau)$, where $\mathcal{M}$ is a von Neumann algebra of bounded operators on some Hilbert space and τ is a normal semifinite faithful trace. The noncommutative L^p spaces associated with $(\mathcal{M}, \tau)$ are denoted $L_p(\mathcal{M})$ and form an interpolation scale which shares many feature with that of the usual L^p spaces. Moreover, whenever $\mathcal{M}$ is commutative, the spaces $L_p(\mathcal{M})$ are classical L^p spaces of functions. Our first line of work in this setting concerns the so-called semicommutative/operator valued functions: fix a von Neumann algebra $\mathcal{M}$ equipped with a trace τ and a measure μ, and define the **semicommutative** trace

$$\varphi(f) = \int_{\mathbb{R}^d} \tau(f(x)) d\mu(x).$$

Denote by $\mathcal{A}$ the algebra of essentially bounded functions $f : \mathbb{R}^d \to \mathcal{M}$. Since $\mathcal{M}$ is not a UMD space, it turns out that the techniques of vector valued Harmonic Analysis cannot be applied and we are therefore forced to use genuinely noncommutative techniques in our study. However, our probabilistic formulation allows us to extend our results on BMO-type spaces to the algebra $\mathcal{A}$.

Theorem 3.1 (C., Mei, Parcet and C.,Parcet [6, 8]). *All the properties of* $\mathrm{BMO}_{\mathrm{ab}}$ *and* $\mathrm{RBMO}_\Sigma(\mu)$ *hold in the setting of $\mathcal{M}$-valued functions.*

Our next contribution concerns the other endpoint of the L_p scale: $p = 1$. It is generally accepted that weak $(1,1)$ inequalities are more difficult to obtain than $L_\infty \to \mathrm{BMO}$ estimates, at least in the noncommutative setting. In particular, we are interested in the following problem:

Problem 3.2. *Let T be a Calderón-Zygmund operator with scalar kernel and bounded on $L^2(\mathcal{A})$. Does T map $L_1(\mathcal{A})$ into $L_{1,\infty}(\mathcal{A})$?*

For the L_1 endpoint problem we have restricted our attention to operators whose structure is well adapted to that of a dyadic lattice on $\mathbb{R}^d$. Namely, we consider the so-called dyadic Hilbert transform, martingale transforms and higher complexity Haar shift operators. For all those dyadic operators T, in [16] the measures μ on $\mathbb{R}^d$ for which $T : L^1(\mu) \to L^{1,\infty}(\mu)$ were characterized. Our contribution, together with López-Sánchez, consists of the semicommutative version of that result. It reads as follows:

Theorem 3.3 (C., López-Sánchez [5])**.** *Let T be a dyadic operator with commutative symbols which is bounded on $L_2(\mathcal{A})$. Then, we have*

- *If T is the dyadic Hilbert transform, then $T : L_1(\mathcal{A} \to L_{1,\infty}(\mathcal{A})$ if and only if μ is m-increasing.*
- *If T is the adjoint of the dyadic Hilbert transform, then $T : L_1(\mathcal{A} \to L_{1,\infty}(\mathcal{A})$ if and only if μ is m-decreasing.*
- *If T is any Haar shift operator of arbitrary complexity and μ is m-equilibrated, then T : $L_1(\mathcal{A} \to L_{1,\infty}(\mathcal{A})$.*

In [6] and [8], we study the relation between BMO function spaces over $\mathbb{R}^d$ and martingale BMO spaces. It turns out that the preduals of martingale BMO spaces, and hence of $\mathrm{BMO}_{\mathrm{ab}}$ and $\mathrm{RBMO}_\Sigma(\mu)$, can be described in terms of some kind of atoms. This is new since the filtrations employed need not be regular. Our result holds in the fully noncommutative case, and in that we formulate it.

Theorem 3.4 (C., Parcet [7])**.** *We say that $b \in L_2(\mathcal{M})$ is a martingale atomic block if there exists an index k such that:*

- $\mathrm{supp}(b) \leq B \in \mathcal{M}_k$,
- $\mathsf{E}_k(b) = 0$,
- $b = \sum_j \lambda_j a_j$. *Each a_j satisfies:*
  - $\mathrm{supp}(a_j) \leq A_j \in \mathcal{M}_{k_j}$, $k_j \geq k$,
  - $\|a_j\|_{L_2(\mathcal{M})} \leq \tau(A_j)^{-\frac{1}{2}}(k_j - k + 1)^{-1}$.

An operator x belongs to the space $\mathrm{H}^1_{\mathrm{atb}}(\mathcal{M})$ if

$$\|x\|_{\mathrm{H}^1_{\mathrm{atb}}} = \inf_{\substack{\mathrm{b}=\sum_j \lambda_j \mathrm{a}_j \\ \mathrm{a}_j \ p-\mathrm{subatom}}} \sum_{i,j} |\lambda_{ij}| < \infty.$$

Then, we have

$$\left[\mathrm{H}^1_{\mathrm{atb}}(\mathcal{M})\right]^* = \mathrm{BMO}(\mathcal{M}).$$

Hence, we have the continuous isomorphism $\mathrm{H}^1_{\mathrm{atb}}(\mathcal{M}) = \mathrm{H}^1(\mathcal{M})$.

The case $0 < p < 1$ is much less understood than the above $p = 1$, both in the martingale setting and in the nonhomogeneous setting, even if we know how to identify the set of functionals acting on the natural definition of $\mathrm{H}^p_{\mathrm{atb}}(\mathcal{M})$.

Problem 3.5. *When $0 < p < 1$, the set of linear bounded functionals acting on $\mathrm{H}^p_{\mathrm{atb}}(\mathcal{M})$ can be identified with the corresponding natural Lipschitz class. Is it indeed true that $\mathrm{H}^p_{\mathrm{atb}}(\mathcal{M}) = \mathrm{H}^p(\mathcal{M})$ as in the case $p = 1$?*

Problem 3.6. *Give a sensible definition of a space* $\mathrm{H}^p_{\mathrm{atb}}(\mathbb{R}^d, \mu)$, $0 < p < 1$.

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